ARO 16539.5-M

ARO Report 83-1

AD A133 627

TRANSACTIONS OF THE TWENTY - EIGHTH

CONFERENCE OF ARMY MATHEMATICIANS

Sponsored by

The Army Mathematics Steering Committee

on behalf of

THE CHIEF OF RESEARCH, DEVELOPMENT

AND ACQUISITION

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER 16539.5-MA	2. GOVT ACCESSION NO. AD-A133 627 N/A	3. RECIPIENT'S CATALOG NUMBER N/A
4. TITLE (and Subtitle) A Transport-Theoretic Analysis of Pulse Propagation Through a Random Cloud of Scatterers		5. TYPE OF REPORT & PERIOD COVERED Reprint
		6. PERFORMING ORG. REPORT NUMBER N/A
7. AUTHOR(s) I. M. Besieris, W. E. Kohler, A. I. Tsolakis		8. CONTRACT OR GRANT NUMBER(s) DAAG29 79 C 0085
9. PERFORMING ORGANIZATION NAME AND ADDRESS Virginia Polytechnic Institute & State University Blacksburg, VA 24061		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS N/A
11. CONTROLLING OFFICE NAME AND ADDRESS U. S. Army Research Office P. O. Box 12211 Research Triangle Park, NC 27709		12. REPORT DATE 1983
		13. NUMBER OF PAGES 10
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Submitted for announcement only.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Accession For	
NTIS GRA&I	☒
DTIC TAB	☐
Unannounced	☐
Justification	
By	
Distribution/	
Availability Codes	
Dist	Avail and/or Special
A	21

DTIC COPY INSPECTED 2

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

83 10 12 230

A TRANSPORT-THEORETIC ANALYSIS OF PULSE PROPAGATION THROUGH A RANDOM CLOUD OF SCATTERERS

I. M. Besieris, W. E. Kohler and A. I. Tsolakis*
Virginia Polytechnic Institute and State University
Blacksburg, VA 24061

ABSTRACT. A systematic development from the Dyson equation and the two-frequency Bethe-Saltpeter equation of a two-frequency radiative transfer equation suitable for pulsed waves in the presence of a random distribution of absorptive discrete scatterers with pair correlations is presented. The main strength of the radiative transfer theory expounded here stems from the fact that it is applicable under conditions of large-angle scattering, statistical inhomogeneities and statistical anisotropies. It accounts, also, for regular refraction (variable scatterer density,) absorption and frequency offsets.

I. INTRODUCTION. Multiple scattering by a random distribution of discrete scatterers has been studied extensively over the past thirty five years, primarily because of its relevance to a large number of pressing applied problems that arise in radio physics and engineering. Fundamental work on multiple scattering of scalar waves by a distribution of uncorrelated scatterers was initiated by Foldy [1] and has been developed further by Lax [2] and Twersky [3]. Based on these original contributions, a great number of applications have appeared in the literature [cf., for example, Refs. 4-7] involving both the coherent field and the incoherent intensity. The problem of assessing the multiple scattering effects on scalar and vector waves in the presence of random distributions of correlated scatterers is more challenging. Basic contributions along this direction have been made by Twersky [8], Bringi *et al*. [9] and Tsang and Kong [10] in connection with the coherent field, and by Barabanenkov [11], Barabanenkov and Finkel'berg [12] and Watson *et al*. [13] in connection with the incoherent field intensity.

The motivation for our work is based on the absence of a second-order statistical theory for studying pulse propagation through random distributions of correlated scatterers. Such a theory is necessary for the development of predictive models pertinent to (1) pulsed electromagnetic propagation through complex natural media (e.g., rain, fog, sandstorms, vegetation); (2) the analysis of obscuration and detection techniques; (3) remote sensing and identification of aerosol clouds; (4) scattering clouds consisting of complicated individual scatterers (lossy, anisotropic, frequency-sensitive, possibly aligned, pair-correlated) with gross macroscopic structure (finite extent and variable number density).

*Present address: Bell Laboratories, Holmdel, NJ 07733.

Our specific goal in this exposition is to derive systematically a radiative transfer equation for the two-frequency incoherent intensity function (a quantity related to the two-frequency mutual coherence function and, hence, to second-order pulse statistics.) The derivation is limited to scalar pulsed waves propagating through a random distribution of absorptive scatterers with pair correlations.

The proposed radiation transport equation is based on the Dyson and Bethe-Saltpeter equations at the level of the direct interaction and ladder approximations, respectively. If, in addition to pair correlations, the assumptions are made that the number of scatterers is large and the average distance between any two scatterers is large compared with a reference wavelength, the Dyson and Bethe-Saltpeter equations are analogous to those associated with a continuous random medium with fluctuations of the permittivity which are distributed according to a normal law and with a deterministic profile directly linked to the number of scatterers per unit volume.

The transition from the Dyson and Bethe-Saltpeter equations to the two-frequency radiative transfer equation is effected by a continuous stochastic transport theory that was originally introduced by Barabanenkov <u>et al</u>. [14] and subsequently extended to the two-frequency context by Besieris and Kohler [15-17]. As in their case, our derivation differs markedly from the usual procedures for obtaining classical radiation transport equations; the latter rely mostly on considerations of energy balance, with no explicit "microscopic" interpretation given to the extinction and scattering coefficients.

In the following we present a sketch of the proposed two-frequency radiation transport theory, with primary emphasis on the underlying assumptions and the physical interpretation of the various terms entering into the final transport equation.

<u>II. FIRST AND SECOND ORDER COHERENCE FUNCTIONS FOR A RANDOM DISTRIBUTION OF PAIR-CORRELATED SCATTERERS, PART A: ANISOTROPIC SCATTERING</u>. The derivation of the Dyson equation for the mean field and the Bethe-Saltpeter equation for the mutual coherence tensor under conditions of anisotropic scattering and in the presence of pair correlations among scatterers is based on the Twersky procedure. The basic underlying assumptions are the following: (1) We ignore third-order scattering by two scatterers, fourth-order scattering by three scatterers, etc. (essentially the Twersky assumption); (2) All scatterers have the same shape, size and orientation distributions; (3) We consider only pair correlations and neglect all contributions from higher-order correlations; (4) The number of scatterers in a volume is infinite.

We resort, also, to the following notational definitions: (1) $\underline{\mathcal{E}}^a$: incident electric field at position $\underline{r}_a$; (2) $\underline{E}^a$: total electric at $\underline{r}_a$;

(3) $g_j{}^a\,\underline{E}^j$: electric field at $\underline{r}_a$ caused by the jth particle; (4) $\langle\langle\cdot\rangle\rangle$: configurational averaging (over size, shape and orientation distributions); (5) $\rho(\underline{r})$: particle density function; (6) $B'(\underline{r}_j, \underline{r}_k)$: pair correlation function.

Under the aforementioned assumptions, the Dyson equation for the coherent vector-valued electric field assumes the form

$$\langle\underline{E}^a\rangle = \underline{\mathcal{E}}^a + \int d\underline{r}_j \, \langle\langle g_j{}^a\rangle\rangle \, \langle\underline{E}^j\rangle \, \rho(\underline{r}_j)$$

$$+ \int d\underline{r}_j \int d\underline{r}_k \, \langle\langle g_j{}^a\rangle\rangle \, \langle\langle G_k{}^j\rangle\rangle \, \langle\underline{E}^k\rangle \, B'(\underline{r}_j, \underline{r}_k) \quad ; \qquad (2.1a)$$

$$\langle\langle G_k{}^a\rangle\rangle \equiv \langle\langle g_k{}^a\rangle\rangle + \int d\underline{r}_\ell \, \langle\langle g_\ell{}^a\rangle\rangle \, \langle\langle G_k{}^\ell\rangle\rangle \, \rho(\underline{r}_\ell)$$

$$+ \int d\underline{r}_\ell \int d\underline{r}_m \, \langle\langle g_\ell{}^a\rangle\rangle \, \langle\langle G_m{}^\ell\rangle\rangle \, \langle\langle G_k{}^m\rangle\rangle \, B'(\underline{r}_\ell, \underline{r}_m) \quad . \qquad (2.1b)$$

On the other hand, the Bethe-Saltpeter equation for the mutual coherence tensor becomes

$$\langle\underline{E}^a\underline{E}^{b*}\rangle = \langle\underline{E}^a\rangle \, \langle\underline{E}^{b*}\rangle + \int d\underline{r}_j \, \langle\langle G_j{}^a\rangle\rangle \, \langle\underline{E}^j \, \underline{E}^{j*}\rangle \, \langle\langle G_j{}^{b*}\rangle\rangle \, \rho(\underline{r}_j)$$

$$+ \int d\underline{r}_j \int d\underline{r}_m \, \langle\langle G_j{}^a\rangle\rangle \, \langle\underline{E}^j \, \underline{E}^{m*}\rangle \, \langle\langle G_m{}^{b*}\rangle\rangle \, B'(\underline{r}_j, \underline{r}_m) \quad . \qquad (2.2)$$

A detailed derivation of Eqs. (2.1) and (2.2) is given in Ref. 18. The former is at the level of the direct interaction approximation, whereas the latter is at the level of the ladder approximation. Both are analogous to equations associated with a continuous random medium with fluctuations of the permittivity which are distributed according to a normal law and with a deterministic refractive profile directly linked to the number of scatterers per unit volume. It should be noted, however, that no counterpart to the second (collapsed) term on the right-hand side of (2.2) exists in the continuous random medium case. In the absence of pair correlations, i.e., $B'(\underline{r}_j, \underline{r}_m) = 0$, (2.1) and (2.2) reduce to the equations derived previously by Twersky [cf. Ref. 19].

The vector-valued Dyson equation (2.1) and the tensor-valued Bethe-Saltpeter equation (2.2) are the basic equations for deriving a tensor-valued radiative transfer equation for vector waves. Such a derivation is very complicated and will not be undertaken in this paper. Instead, Eqs. (2.1) and (2.2) will be "scalarized" in the next section. (A set of assumptions sufficient for such an approximation is given in the Appendix.) The resulting equations will form the basis for deriving a scalar radiation transport theory in Sec. IV.

III. FIRST AND SECOND ORDER COHERENCE FUNCTIONS FOR A RANDOM DISTRIBUTION OF PAIR-CORRELATED SCATTERERS, PART B: ISOTROPIC SCATTERING.

In the following, we shall assume that the scattering channel is tenuous, i.e., the distance between any two scatterers is much greater than a reference wavelength. We shall also assume scalar isotropic scattering. In the electromagnetic case this approximation arises if individual scatterer dimensions are small compared to wavelength and if "gross" depolarization effects are neglected. In the case of acoustic wave propagation, isotropic scattering takes place when individual scatterer dimensions are small compared to wavelength.

Under these assumptions, one deals, essentially, with a scalar wave theory. The Dyson equation (2.1) simplifies considerably and is rewritten below in a form suitable for our work in Sec. IV:

$$[\nabla^2 + k^2 + 4\pi f\, \rho(\underline{r})]\, \langle E(\underline{r}, k)\rangle$$

$$= -4\pi f \int d\underline{r}'\, \langle\langle G'(\underline{r}, \underline{r}', k)\rangle\rangle\, \langle E(\underline{r}', k)\rangle\, B'(\underline{r}, \underline{r}') \quad ; \qquad (3.1a)$$

$$[\nabla^2 + k^2 + 4\pi f\, \rho(\underline{r})]\, \langle\langle G'(\underline{r}, \underline{r}', k)\rangle\rangle$$

$$= -4\pi f\, \delta(\underline{r} - \underline{r}') - 4\pi f \int d\underline{r}''\, \langle\langle G'(\underline{r}, \underline{r}'', k)\rangle\rangle\, \langle\langle G'(\underline{r}'', \underline{r}', k)\rangle\rangle\, B'(\underline{r}, \underline{r}'') \, . \qquad (3.1b)$$

The notation is identical to the one used in the previous section, except that E and G' are now scalar-valued. The quantities k and f are respectively the wavenumber and a configurationally averaged scalar-valued scattering coefficient. The latter is independent of $\underline{r}$; however, it may depend on k and, in general, is complex by virtue of the absorptive properties of the scatterers.

The scalar-valued Bethe-Saltpeter equation for the two-frequency mutual coherence function $\Gamma'(\underline{r}_1, \underline{r}_2, k_1, k_2) \equiv \langle E(\underline{r}_1, k_1)E^*(\underline{r}_2, k_2)\rangle$ can be written as follows:

$$[(\nabla_{\underline{r}_1}^{\;2} - \nabla_{\underline{r}_2}^{\;2}) + (k_1^{\,2} - k_2^{\,2}) + 4\pi f\, \rho(\underline{r}_1) - 4\pi f^*\, \rho(\underline{r}_2)]\, \Gamma'(\underline{r}_1, \underline{r}_2, k_1, k_2)$$

$$= -4\pi\{\rho(\underline{r}_1) f\, \langle\langle G'^*(\underline{r}_2, \underline{r}_1, k_2)\rangle\rangle\, \Gamma'(\underline{r}_1, \underline{r}_1, k_2)$$

$$-\rho(\underline{r}_2) f^*\, \langle\langle G'(\underline{r}_1, \underline{r}_2, k_1)\rangle\rangle\, \Gamma'(\underline{r}_2, \underline{r}_2, k_1, k_2)\}$$

$$-4\pi \int d\underline{r}' \, [f\, B'(\underline{r}_1, \underline{r}') - f^* B'(\underline{r}', \underline{r}_2)][\langle\langle G'(\underline{r}_1, \underline{r}', k_1)\rangle\rangle \, \Gamma'(\underline{r}', \underline{r}_2, k_1, k_2)$$

$$+ \langle\langle G'^*(\underline{r}_2, \underline{r}', k_2)\rangle\rangle \, \Gamma'(\underline{r}_1, \underline{r}', k_1, k_2)] \quad . \tag{3.2}$$

The derivation of (3.2) [cf. Ref. 18] incorporates a narrowband pulse assumption. In this case, the scattering coefficient f depends only on the prescribed carrier frequency.

The functional forms of Eqs. (3.1) and (3.2) are analogous to those derived by Besieris and Kohler [15,16] for acoustic wave propagation in a continuous random medium characterized by Gaussian fluctuations. As in the case of the more general equation (2.2), no counterpart of the first (collapsed) term on the right-hand side of (3.2) exists in the continuous random case. For $k_1 = k_2$ and in the absence of pair correlations, i.e., $B'(\underline{r}_1, \underline{r}_2) = 0$, Eqs. (3.1) and (3.2) reduce to relationships already available in the literature [cf. Ref. 19].

<u>IV. TWO-FREQUENCY RADIATIVE TRANSFER EQUATION FOR A RANDOM DISTRIBUTION OF ABSORPTIVE SCATTERERS WITH PAIR CORRELATIONS: SCALAR WAVE THEORY</u>. In the Bethe-Saltpeter equation (3.2) for $\Gamma'(\underline{r}_1, \underline{r}_2, k_1, k_2)$ we introduce center-of-mass and difference coordinates and wavenumbers, viz., $\underline{R} \equiv (\underline{r}_1 + \underline{r}_2)/2$, $\underline{r} \equiv \underline{r}_1 - \underline{r}_2$; $k_s \equiv (k_1 + k_2)/2$, $k_d \equiv k_1 - k_2$, and use the notational definitions $\Gamma'(\underline{r}_1, \underline{r}_2, k_1, k_2) \equiv \Gamma(\underline{R}, \underline{r}, k_s, k_d)$, $B'(\underline{r}_1, \underline{r}_2) \equiv B(\underline{R}, \underline{r})$, $\langle\langle G'(\underline{r}_1, \underline{r}_2, k)\rangle\rangle \equiv \langle\langle G(\underline{R}, \underline{r}, k)\rangle\rangle$, and $M(\underline{R}, \underline{r}, k) \equiv f\langle\langle G(\underline{R}, \underline{r}, k)\rangle\rangle \, B(\underline{R}, \underline{r})$. We introduce, also, the Fourier transform pairs $\Gamma(\underline{R}, \underline{r}, k_s, k_d) \longleftrightarrow f(\underline{R}, \underline{\kappa}, k_s, k_d)$, $B(\underline{R}, \underline{r}) \longleftrightarrow \Phi(\underline{R}, \underline{\kappa})$, and $M(\underline{R}, \underline{r}, k) \longleftrightarrow \tilde{M}(\underline{R}, \underline{\kappa}, k) = \tilde{M}'(\underline{R}, \underline{\kappa}, k) + i\, \tilde{M}''(\underline{R}, \underline{\kappa}, k)$. It should be noted that $M(\underline{R}, \underline{r}, k)$ is analogous to the "mass operator" entering into the Dyson equation in the case of smoothly inhomogeneous media. Furthermore, the quantity $f(\underline{R}, \underline{\kappa}, k_s, k_d)$ is the two-frequency extension to the phase-space Wigner distribution function.

We consider next smoothly inhomogeneous media for which $\Gamma(\underline{R}, \underline{r}, k_s, k_d)$, $B(\underline{R}, \underline{r})$ and $\langle\langle G(\underline{R}, \underline{r}, k)\rangle\rangle$ vary slowly with respect to the sum variable $\underline{R}$, and rapidly with respect to the difference variable $\underline{r}$. We make also two further assumptions: (1) the ratio of difference to sum wavenumbers is small compared to unity, i.e., $|k_d/k_s| \ll 1$; (2) the scattering and regular losses are small but not negligible. Within the framework of these restrictions scattering becomes significant [cf. Ref. 18 for details] on the "energy" surface

$$H'(\underline{R}, \underline{\kappa}, k_s) \equiv \frac{1}{2}[\kappa^2 - k_s^2 - 4\pi f_R \, \rho(\underline{R}) - 4\pi \, \tilde{M}'(\underline{R}, \underline{\kappa}, k_s)] = 0 \quad , \tag{4.1}$$

which is independent of regular and scattering losses. The quantity f_R in (4.1) denotes the real part of the complex scattering coefficient f.

In the general case of statistically anisotropic pair correlations of the scatterers we seek a solution in the form

$$f(\underline{R}, \underline{\kappa}, k_s, k_d) = f_o(\underline{R}, \underline{\kappa}, k_s, k_d) + f_1(\underline{R}, \underline{\kappa}, k_s, k_d) \quad . \tag{4.2}$$

The first term on the right-hand side of (4.2) is the coherent part of the two-frequency Wigner distribution and is directly related to the solution of the Dyson equation (3.1). The incoherent part of the Wigner density function, on the other hand, is chosen as follows:

$$f_1(\underline{R}, \underline{\kappa}, k_s, k_d) = k_s |\nabla_{\underline{\kappa}} H'(\underline{R}, \underline{\kappa}, k_s)| \, |\nabla_\kappa H'(\underline{R}, \underline{\kappa}, k_s)|^{-3}$$

$$\times \, \delta[H'(\underline{R}, \underline{\kappa}, k_s)] \, I(\underline{R}, \underline{s}, k_s, k_d) \quad ; \quad \underline{s} = \underline{\kappa}/\kappa \quad . \tag{4.3}$$

The quantity $I(\underline{R}, \underline{s}, k_s, k_d)$ is the two-frequency incoherent "ray" intensity at the point $\underline{R}$ and in the direction of the unit vector $\underline{s}$.

Let, next, $k_{eff}(\underline{R}, \underline{s}, k_s)$ denote the value of κ for which $H'(\underline{R}, \kappa\underline{s}, k_s) = 0$, and define an effective index of refraction as follows:

$$n_{eff}(\underline{R}, \underline{s}, k_s) = |\nabla_\kappa H'(\underline{R}, \kappa\underline{s}, k_s)|/k_s \quad ; \quad \kappa = k_{eff}(\underline{R}, \underline{s}, k_s) \quad . \tag{4.4}$$

Let, finally, $\Theta(\underline{R}, \underline{s}, k_s)$ be the angle between the direction of the group and phase velocities. With these definitions in mind, the incoherent ray intensity $I(\underline{R}, \underline{s}, k_s, k_d)$ is found to obey the two-frequency radiative transfer equation

$$n^2_{eff}(\underline{R}, \underline{s}, k_s) \frac{d}{dl} \{I(\underline{R}, \underline{s}, k_s, k_d)|\cos \Theta(\underline{R}, \underline{s}, k_s)|^{-1} n^{-2}_{eff}(\underline{R}, \underline{s}, k_s)\}$$

$$= \{-4 \, f_I \, \rho(\underline{R}) - 4\pi \, \tilde{M}''[\underline{R}, k_{eff}(\underline{R}, \underline{s}, k_s)\underline{s}, k_s] + i \, \frac{1}{4} k_s k_d\}$$

$$\times \, k_s^{-1} \, n^{-1}_{eff} (\underline{R}, \underline{s}, k_s) \, I(\underline{R}, \underline{s}, k_d)$$

$$+ \, 2|f|^2 \, k_s^{-2} \int_\Omega d\underline{s}' \, k^2_{eff} (\underline{R}, \underline{s}', k_s) n_{eff}(\underline{R}, \underline{s}, k_s) n^{-3}_{eff}(\underline{R}, \underline{s}', k_s)$$

$$\times \, |\cos\Theta(\underline{R}, \underline{s}', k_s)|^{-1}\{\Phi[\underline{R}, k_{eff}(\underline{R}, \underline{s}, k_s)\underline{s} - k_{eff}(\underline{R}, \underline{s}, k_s)\underline{s}'] + \rho(\underline{R})\}$$

$$\times \, I(\underline{R}, \underline{s}', k_s, k_d)$$

$$+ \, 2|f|^2 \, n_{eff}(\underline{R}, \underline{s}, k_s)|\cos\Theta(\underline{R}, \underline{s}, k_s)| \int_{R^3} d\underline{\kappa}' \, \{\Phi[\underline{R} - k_{eff}(\underline{R}, \underline{s}, k_s)\underline{s} - \underline{\kappa}']$$

$$+ \, \rho(\underline{R})\} \, f_o(\underline{R}, \underline{\kappa}', k_s, k_d) \quad , \tag{4.5}$$

where Ω denotes the range of $\underline{s}'$ over the surface of a unit sphere.

Equation (4.5) for the two-frequency incoherent intensity $I(\underline{R}, \underline{s}, k_s, k_d)$ is the main result of this paper. In interpreting this equation, we should note the following: The left-hand side of the equation is a convective term; the ray paths correspond to an effective medium determined by the density of the scatterers $\rho(\underline{R})$, the scattering coefficient f and the spatial correlation function of the scatterers. [The quantity dl denotes the differential of a curvilinear ray passing through the point $\underline{R}$ in the direction $\nabla_{\kappa} H'(\underline{R}, \underline{\kappa}, k_s)$.] The first and second terms on the right-hand side of (4.5) are due respectively to regular and scattering losses; the third term arises because of frequency offsets; the fourth one is the scattering term; finally, the last one is the source term, representing the "feeding" of the incoherent intensity by the coherent part of the Wigner distribution function.

In order to compute the second-order pulse moment $\langle E(\underline{r}_1, t_1)E^*(\underline{r}_2, t_2)\rangle$ at the receiver site, we must first find the two-frequency mutual coherence function $E(\underline{r}_1, \omega_1)E^*(\underline{r}_2, \omega_2)$ and then perform a two-dimensional Fourier transform with respect to ω_1 and ω_2. In the ladder approximation, the two-frequency mutual coherence function obeys the Bethe-Saltpeter equation (3.2). The phase-space analog of the two-frequency mutual coherence function is the Wigner distribution $f(\underline{R}, \underline{\kappa}, k_s, k_d)$. The latter is decomposed in (4.2) into a coherent part, which is directly linked to the solution of the Dyson equation (3.1), and an incoherent part directly expressible [cf. Eq. (4.3)] in terms of the two-frequency ray intensity $I(\underline{R}, \underline{s}, k_s, k_d)$. The latter obeys the radiative transfer equation (4.5).

Not all the successive steps outlined in the previous paragraph need be followed for obtaining information about second-order pulse statistics. If, for example, such information is restricted to the incoherent part of $\langle E(\underline{r}_1, t_1)E^*(\underline{r}_2, t_2)\rangle$, direct usage can be made of the two-frequency ray intensity $I(\underline{R}, \underline{s}, k_s, k_d)$, as explained below.

Let $\Gamma_i(\underline{R}, \underline{\kappa}, k_s, k_d) \longleftrightarrow f_i(\underline{R}, \underline{\kappa}, k_s, k_d)$ denote the incoherent part of $\Gamma(\underline{R}, \underline{\kappa}, k_s, k_d)$. Then,

$$\Gamma_i(\underline{R}, \underline{r}, k_s, k_d) = \int_\Omega d\underline{s}\, k_s^{-2}\, n_{eff}^{-3}(\underline{R}, \underline{s}, k_s) k_{eff}^2(\underline{R}, \underline{s}, k_s)$$

$$\times\, |\cos\theta(\underline{R}, \underline{s}, k_s)|^{-1}\, I(\underline{R}, \underline{s}, k_s, k_d) \exp\{i\, k_{eff}(\underline{R}, \underline{s}, k_s)\underline{s} \cdot \underline{r}\} \quad , \qquad (4.6)$$

which establishes a useful connection between the photometric ray intensity $I(\underline{R}, \underline{s}, k_s, k_d)$ and the incoherent part of the two-frequency mutual coherence function.

V. CONCLUDING REMARKS. A radiative transfer equation for pulsed scalar waves in a random distribution of pair correlated absorptive scatterers has been derived systematically from the Dyson and Bethe-Saltpeter equations. Detailed solutions -- both analytical and numerical -- are presently under consideration. An important question in this context is whether controlled experiments could be carried out so that comparisons would be made with theoretical results.

An open research area in propagation through random distributions of scatterers is the systematic derivation of a tensor transport theory from the Dyson equation (2.1) and the Bethe-Saltpeter equation (2.2). Such a theory is of paramount importance for physical situations where anisotropic scattering and depolarization effects cannot be neglected.

ACKNOWLEDGMENTS. Research supported in part by the Army Research Office under contract No. DAAG 29-79-C-0085 and the Office of Naval Research under contract No. N00014-76-C-0056.

APPENDIX. Consider the case of Born approximation valid for single particle scattering together with the simplification

$$\int_{V_s} d\underline{r}'[\epsilon_r(\underline{r}') - 1] \exp\{ik(\hat{\underline{i}} - \hat{\underline{o}}) \cdot \underline{r}'\} \approx \int_{V_s} d\underline{r}'[\epsilon_r(\underline{r}') - 1] \quad . \qquad (A-1)$$

In this expression, V_s denotes the volume of the scatterer, $\epsilon_r(\underline{r})$ is the relative permittivity of the scatterer, and $\hat{\underline{i}}$, $\hat{\underline{o}}$ are respectively unit vectors along the incident and scattered directions. In this case, the tensor-valued quantity $\langle\langle g_j^a \rangle\rangle$ obeys the equation

$$[\nabla_{\underline{r}_a} \times \nabla_{\underline{r}_a} \times (\cdot) - k^2 I] \langle\langle g_j^a \rangle\rangle = k^2\, \xi\, \delta(\underline{r}_a - \underline{r}_j) I \quad , \qquad (A-2)$$

where

$$\xi = \xi' + i\,\xi'' = \langle\langle \int_{V_s} d\underline{r}' [\varepsilon_r(\underline{r}') - 1] \rangle\rangle \tag{A-3}$$

and I is the unit tensor (dyadic).

Let

$$M^a(\cdot) = [\nabla_{\underline{r}_a} \times \nabla_{\underline{r}_a} \times (\cdot) - k^2(1 + \xi\, \rho(\underline{r}_a))I] \quad . \tag{A-4}$$

Then, the Dyson equation (2.1) reduces to

$$M^a \langle \underline{E}^a \rangle = k^2\xi \int d\underline{r}_k \quad \langle\langle G_k{}^a \rangle\rangle \ \langle \underline{E}^k \rangle \ B'(\underline{r}_a, \underline{r}_k) \quad ; \tag{A-5a}$$

$$M^a \langle\langle G_j{}^a \rangle\rangle = k^2\xi \ \delta(\underline{r}_a - \underline{r}_j)I$$

$$+ k^2\xi \int d\underline{r}_m \ \langle\langle G_m{}^a \rangle\rangle \ \langle\langle G_j{}^m \rangle\rangle \ B'(\underline{r}_a, \underline{r}_m) \quad . \tag{A-5b}$$

The Bethe-Saltpeter equation (2.2) simplifies to

$$M^a \langle \underline{E}^a \ \underline{E}^{b*} \rangle - \langle \underline{E}^a \ \underline{E}^{b*} \rangle \ M^{b*}$$

$$= k_1{}^2\xi \int d\underline{r}_j \ \langle\langle G_j{}^a \rangle\rangle \ \langle \underline{E}^j \ \underline{E}^{b*} \rangle \ B'(\underline{r}_a, \underline{r}_j)$$

$$- k_2{}^2\xi^* \int d\underline{r}_j \ \langle \underline{E}^a \ \underline{E}^{j*} \rangle \ \ \langle\langle G_j{}^{b*} \rangle\rangle \ B'(\underline{r}_j, \underline{r}_b)$$

$$+ k_1{}^2\xi \int d\underline{r}_j \ \langle \underline{E}^a \ \underline{E}^{j*} \rangle \ \langle\langle G_j{}^{b*} \rangle\rangle \ B'(\underline{r}_a, r_j)$$

$$- k_2{}^2\xi^* \int d\underline{r}_j \ \langle\langle G_j{}^a \rangle\rangle \ \langle \underline{E}^j \ \underline{E}^{b*} \rangle \ B'(\underline{r}_j, r_b)$$

$$+ k_1{}^2\xi \ \langle \underline{E}^a \ \underline{E}^{a*} \rangle \ \langle\langle G_a{}^{b*} \rangle\rangle - k_2{}^2\xi^* \ \langle\langle G_b{}^a \rangle\rangle \ \langle \underline{E}^b \ \underline{E}^{b*} \rangle \quad . \tag{A-6}$$

If macroscopic depolarization effects are neglected, that is, if the approximation $\nabla\times\nabla\times(\cdot) = \nabla[\nabla \cdot (\cdot)] - \nabla^2 I \simeq \nabla^2 I$ can be justified, both equations (A-5) and (A-6) reduce to scalar relationships.

REFERENCES

1. L. O. Foldy, Phys. Rev. 67, 107 (1945).
2. M. Lax, Rev. Mod. Phys. 23, 287 (1951).
3. V. Twersky, Proc. Am. Math. Soc. 16, 84 (1969).
4. A. Ishimaru, Proc. IEEE 65, 1030 (1977).
5. K. Furutsu, J. Math. Phys. 21, 765, 2764 (1980); J. Opt. Soc. Am. 70, 360 (1980).
6. S. Ito and K. Furutsu, J. Opt. Soc. Am. 70, 365 (1980).
7. S. Ito, Appl. Opt. 20, 2706 (1981).
8. V. Twersky, J. Math. Phys. 19, 215 (1978).
9. V. N. Bringi, T. A. Seliga, V. K. Varadan and V. V. Varadan, "Bulk Propagation Characteristics of Discrete Random Media," in Multiple Scattering and Waves in Random Media, edited by P. L. Chow, W. E. Kohler and G. C. Papanicolaou (North-Holland Publishing Co., Amsterdam, 1981).
10. L. Tsang and J. A. Kong, J. Appl. Phys. 52, 5448 (1981).
11. Yu. N. Barabanenkov, Sov. Phys. - Doklady 12, 431 (1967).
12. Yu. N. Barabanenkov and V. M. Finkel'berg, Sov. Phys. JETP 26, 587 (1968).
13. K. M. Watson, J. Math. Phys. 10, 688 (1969); C. W. Law and K. M. Watson, J. Math. Phys. 11, 3125 (1970).
14. Yu. N. Barabanenkov, A. G. Vinogradov, Yu. A. Kravtsov and V. I. Tatarskii, Izv. VUZ., Radiofiz. 15, 1852 (1972).
15. I. M. Besieris and W. E. Kohler, "Derivation of the Two-Frequency Radiative Transfer Equation for a Statistically Innomogeneous and Anisotropic Medium," Proc. International U.R.S.I. Symp., Munich, Germany (1980).
16. I. M. Besieris and W. E. Kohler, "Two-Frequency Radiative Transfer Equation for a Statistically Inhomogeneous and Anisotropic Absorptive Medium," in Multiple Scattering and Waves in Random Media, edited by P. L. Chow, W. E. Kohler and G. C. Papanicolaou (North-Holland Publishing Co., Amsterdam, 1981).
17. I. M. Besieris, W. E. Kohler and H. Freese, "A Transport-Theoretic Analysis of Pulse Propagation Through Ocean Sediments," J. Acoust. Soc. Am. (to appear).
18. A. I. Tsolakis, "Multiple Scattering of Electromagnetic Waves by Distributions of Particles with Applications to Radio Wave Propagation Through Precipitation," Ph.D. Thesis, Virginia Polytechnic Institute and State University.
19. A. Ishimaru, Wave Propagation and Scattering in Random Media (Academic Press, New York, N.Y., 1978).